Estimation of Rolling Process Stability by Contact-Stress Diagrams

O. P. Maksimenko, R. Ya. Romanyuk

Dniproderzhynsk State Technical University
2 Dneprostroevska St., Dniproderzhynsk, Ukraine

An average resultant of horizontal forces in the deformation zone that defines stability of rolling process was determined as a result of analysis of experimental contact-stress diagrams. Its value is considerable under stable process and closely related to kinematic parameters, in particular, forward slip. This resultant includes a constituent related to metal strain and dynamic constituent appearing when system of particles accelerates in the area of strip and rolls contact.

Keywords: FORWARD SLIP, CONTACT STRESSES, DISTRIBUTION DIAGRAM, RESULTANT OF HORIZONTAL FORCES, DEFORMATION ZONE, DYNAMIC FORCE, STABLE PROCESS

Introduction

It is accepted in theory that angle of neutral cross-section is a regulator of force balance in the deformation zone. And there are extreme conditions of rolling at this angle equal zero.

At the same time, there are known experimental data of stable process at one-zone metal forward in rolls (at "negative" forward flow), which is difficult to explain in view of current theory.

A. P. Grudev's experiments [1] show that rolling process can be discontinued at considerable forward flow. These results also were not explained.

In [2-4] it is shown on the basis of analysis of theoretical contact-stress diagrams that the average resultant of horizontal forces in deformation zone can be as an indicator of process stability.

Depending on value and direction of this force, rolling is either stable without slip or carried on under extreme conditions or impossible.

The purpose of work is further study of regularities of change of average resultant of horizontal forces in deformation zone by experimental contact-stress diagrams, quantitative estimation and analysis of this force at stable and limiting rolling.

Results and Discussion

Contact stress distribution experimental data presented in [5, 6] were used. It is necessary to notice that the normal pressure and specific friction forces distribution diagrams were selected with account of various kinematic conditions providing rolling with metal forward flow or at one-zone forward in the deformation zone. Rolling conditions are presented in Table 1.

Pressure profiles $p_x$ and specific friction force distribution diagrams $t_x$ at hot rolling of steel samples with shape parameter $\frac{1d}{h_{av}} = 2.7$ given in [5] are presented in Figure 1. The change of longitudinal stresses $q_x$ and current resultant of horizontal forces $Q_x$ calculated by formulas (1) and (2) respectively is presented as follows:

$$q_x = -p_x \sin \varphi + t_x \cos \varphi, \quad (Eq. 1)$$

$$Q_x = R \cdot b \int_{\varphi}^{\alpha} q_x d\varphi, \quad (Eq. 2)$$

where $\varphi$ – the current angle in the deformation zone; $R$ – the roll radius.
Figure 1. Distribution of contact stresses and longitudinal resultant of horizontal forces lengthwise the deformation zone when steel samples rolling
Table 1. Rolling parameters at contact stress investigation

<table>
<thead>
<tr>
<th>No.</th>
<th>Sample thickness, mm</th>
<th>Average friction coefficient ( f )</th>
<th>Roll diameter ( D ), mm</th>
<th>Angle of nip ( \alpha ), rad</th>
<th>Shape parameter ( \frac{I_d}{h_{av}} )</th>
<th>Source</th>
<th>Rolling conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6 6 4.2</td>
<td>0.207</td>
<td>210</td>
<td>0.131</td>
<td>2.7</td>
<td>[4]</td>
<td>Hot rolling of steel</td>
</tr>
<tr>
<td>2</td>
<td>2 1</td>
<td>0.123</td>
<td>208</td>
<td>0.098</td>
<td>7.0</td>
<td>[5]</td>
<td>Plumbum rolling with process lubricant</td>
</tr>
<tr>
<td>3</td>
<td>2.2 1.2</td>
<td>0.048</td>
<td>208</td>
<td>0.098</td>
<td>6.0</td>
<td>[5]</td>
<td>Plumbum rolling with process lubricant</td>
</tr>
</tbody>
</table>

*Note: sample width \( b = 50 \) mm

As follows from the graph, current force \( Q_x \) in all cross-sections of strip contact with rolls is positive and it is zero on boundaries of deformation zone. From the latter it follows that steady-state process of rolling is being considered. The average value of this force defined by formula:

\[
Q_{av} = \frac{1}{\alpha_y} \int_{0}^{\alpha_y} Q_x d\varphi,
\]  

(Eq. 3)

is also positive and equals 10.19 kN.

According to [3], rolling process should progress stably without slips at such value of average horizontal force (as contact stress diagrams show).

Graphs in Figure 2 (solid line) show contact stress diagrams given in [6] and also diagrams of longitudinal stresses and current resultant force \( Q_x \) distribution for conditions of the second experiment (Table 1).

As compared to previous case, specific friction forces were weaker, forward zone was smaller, which affected the current resultant force distribution. It had a negative value in the initial part of deformation zone. In the middle part and towards metal exit from the rolls, this force changed a sign and its vector coincided with strip motion direction. An average value of this force was \( Q_{av} = 0.085 \) kN. Results of experiments confirm stability of rolling process.

The third experiment data shown in Figure 2 (dotted line) characterize rolling with one-zone forward of metal. The author [6] marks that in this case the diagrams were obtained at unstable rolling with metal slip. Analyzing longitudinal stress curve \( q_x \), we can notice that its change is opposite to cases considered above. Force \( Q_x \) distribution zone has a negative value, and average resultant \( Q_{av} \) brakes strip motion \( Q_{av} = -0.101 \) kN. Force \( Q_{av} \) is 8% from deformation force for conditions of the first experiment at stable rolling process. It is obvious that in this case total force is not directed vertically and inclined towards the strip motion.

Thus, both theoretical investigations [2-4] and analysis of experimental contact stress diagrams show that the average resultant of horizontal forces depending on rolling parameters can accept different values and characterize extreme conditions of deformation.

Force \( Q_{av} \) obtained on the basis of processing experimental contact stress diagrams includes constituents related to metal flow and acceleration of metal particles in the deformation zone.

We will estimate a dynamic constituents of average resultant force \( Q_{av} \). For this purpose we will use momentum theorem for dynamic system of material points [7], according to which time derivative from system momentum vector equals the resultant vector of external forces applied to this system, i.e.

\[
\frac{dk}{dt} = \sum_{i=1}^{n} F_i,
\]  

(Eq. 4)

where \( k \) – the system momentum, \( k = \sum_{i=1}^{n} m_i V_i \);

\( F_i \) – the force acting on the given point of material system; \( m_i \) – the material particle mass; \( V_i \) – the speed of given particle.
Figure 2. Distribution of contact stresses and longitudinal resultant of horizontal forces lengthwise the deformation zone when plumbous samples rolling.
Having applied mentioned above theorem for system of material particles of deformation zone and assuming that in the first approximation \( \sum_{i=1}^{n} F_i = F_{av} \), we will obtain:

\[
M(V_1 - V_0) = F_{av} t,
\]

(Eq. 5)

where \( M(V_1 - V_0) \) – the momentum change when mass metal \( M \) goes through the deformation zone; \( V_0 \) and \( V_1 \) – the metal rate at the entry in rolls and exit from them respectively; \( t \) – the time during which sample mass \( M \) will go through metal flow zone; \( F_{av} \) – the average value of resultant of external dynamic forces applied to system of particles of deformation zone.

Considering that acceleration of material particles occurs in the deformation zone system, we can define \( F_{av} \) value from Equation 5.

Results of calculation of average values of resultants of horizontal forces \( Q_{av} \) and \( F_{av} \) for steel hot rolling are presented in Table 2.

Analysis of calculation results shows that dynamic component \( F_{av} \) is much less than force \( Q_{av} \) and their ratio is approximately \( \frac{F_{av}}{Q_{av}} = 0.01 \).

In this case, force \( Q_{av} \) should be balanced by plastic deformation constituent.

We will calculate theoretical value of resultant \( Q_{av} \) using technique [4] under first experimental rolling conditions (Table 1) and coefficient of friction \( f = 0.207 \). Results of calculation are presented in Figure 3. Non-dimensional current resultant of horizontal forces \( Q^*_x \) changes in the same way as when analyzing experimental distribution diagrams (Figure 1). Its average value is:

\[
Q^*_{av} = \frac{Q_{av}}{Rb\beta\sigma_Y S} = 0.0096.
\]

(Eq. 6)

We will find theoretical value of force \( Q_{av} \).

If proceed from the maximum value of specific friction forces in the first experiment and theoretical graph of specific friction force distribution (Figure 3), it is possible to derive an approximate equality: \( 0.5\beta\sigma_Y S = 90 \text{ MPa} \).

Then theoretical value of average resultant force is \( Q_{av} = 9.07 \text{ kN} \).

Though results of processing of experimental contact-stress diagrams and theoretical calculation differ (10.19 kN and 9.07 kN) it is possible to conclude that force \( Q_{av} \) is determined primarily by plastic component since Karman’s equation does not consider dynamics of the process.

Besides, according to differential equilibrium equation, resultant \( Q_{av} \) can be balanced by only an average resultant of internal forces \( Q_{av ext} \) from action of longitudinal stresses. Having a theoretical pressure profile (Figure 3) and considering plasticity equation it is easy to define a current value of direct internal forces:

\[
Q_x = \sigma_x h_x b = (p_x - \beta\sigma_Y S) h_x b, \quad (Eq. 7)
\]

\[
Q_{av ext} = \frac{1}{\alpha_y} \int_0^{\alpha_y} Q_x \text{d}\varphi, \quad (Eq. 8)
\]

It is obvious that we will obtain the following equation as a result of calculation:

\[
Q_{av ext} = 2Q_{av}, \quad (Eq. 9)
\]

Table 2. Results of calculation of forces \( Q_{av} \) and \( F_{av} \)

<table>
<thead>
<tr>
<th>Experimental data</th>
<th>Density of sample material ( \rho ), t/m³</th>
<th>Metal mass in deformation zone ( M ), kg</th>
<th>Length of sample mass ( M ) after rolling, ( l_1 ), mm</th>
<th>Rolling rate ( v_\theta ), m/s</th>
<th>Force resultants, kN</th>
</tr>
</thead>
<tbody>
<tr>
<td>[3]</td>
<td>7.85</td>
<td>0.0274</td>
<td>16.6</td>
<td>0.5</td>
<td>10.19</td>
</tr>
</tbody>
</table>

Figure 3. Distribution of non-dimensional stresses and force $Q_x^*$ lengthwise the deformation zone at theoretical investigation of steel samples rolling.
Consequently, the external resultant force $Q_{av}$ is balanced by internal direct forces. In addition, the sum of internal forces is equal to zero in deformation zone. It follows from boundary conditions of simple rolling process: longitudinal stresses are equal to zero at the entry and exit from deformation zone. Non-dimensional average resultant of horizontal forces $Q^*_av$ is closely related to kinematic parameters of rolling and, in particular, metal forward, which can be seen from graphs in Figure 4. Theoretical calculation of forward flow was carried out by known Golovin’s formula. The technique presented in [4] was used to determine force $Q^*_av$. Experimental values of forward flow and rolling conditions were taken from A. P. Grudev’s work [1]: roll diameter D=194.6 mm, final thickness of samples $h_1=1.1$ mm, friction coefficient $f_y=0.25$. As shown in Figure 4, dependence of forward and force $Q^*_av$ on angle of nip $\alpha_y$ is qualitatively similar. Experimental and calculated values of forward are very close. It is important to note that according to author [1] there are boundary conditions of rolling at angle of nip $\alpha_y=0.34$ rad though forward is 10.1%. It follows from graph (Figure 4, b) that non-dimensional resultant force $Q^*_av$ is equal to zero. Presented graphs confirm that force $Q^*_av$ shows the real stability of rolling and indicates the possibility of metal slipping in the deformation zone.

![Diagram](image)

**Figure 4.** Dependence of forward flow and force $Q^*_av$ on angle of nip: a - A. P. Grudev’s practical data; b - results of theoretical investigation

**Conclusions**

It is possible to define an average resultant of horizontal forces in the deformation zone when analyzing experimental contact stress diagrams as well as in theory. Depending on experimental rolling conditions it can have different values. At steady rolling this force is positive and considerable. Boundary conditions of rolling occur at its zero value.

At steady rolling process, the total force of all external forces in the deformation zone is not directed vertically but inclined towards the strip motion. Dynamic component $Q_{av}$ is rather small under experimental rolling conditions. In general, the average resultant of horizontal forces is balanced by plastic component related to longitudinal normal stresses in the deformation zone.

One of areas of suggested technology application is specification of roll bite conditions in steady rolling process.

**References**

Оценка устойчивости процесса прокатки по эпюрам контактных напряжений

Максименко О.П., Романюк Р.Я.

В результате анализа опытных эпюр контактных напряжений выделена средняя результирующая горизонтальных сил в очаге деформации, которая определяет устойчивость процесса прокатки. Она значительна по величине при устойчивом процессе, тесно связана с кинематическими параметрами, в частности, с опережением и включает в себя составляющую, связанную с деформацией металла, и динамическую составляющую, возникающую при ускорении системы материальных частиц в зоне контакта полосы с валками. Динамическая составляющая при этом существенно меньше результирующей горизонтальных сил.